THE VILLAGE
INTERNATIONAL SCHOOL
"We Nurture Dreams"

CLASS12 DEPARTMENT OF MATHEMATICS

MATRICES & DETERMINANTS

ONE MARK QUESTIONS

1. If $[1\ x\ 1]\begin{bmatrix}1 & 3 & 2\\ 0 & 5 & 1\\ 0 & 3 & 2\end{bmatrix}\begin{bmatrix}x\\ 1\\ -2\end{bmatrix} = 0$, then What is the value of x?

2. For what value of λ, the matrix A is a singular matrix where

$$A = \begin{bmatrix}1 & 3 & \lambda+2\\ 2 & 4 & 8\\ 3 & 5 & 10\end{bmatrix}$$

3. Find the value of A^2, if

$$A = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ a & b & -1\end{bmatrix}$$

4. If $A = \begin{bmatrix}a & b\\ b & a\end{bmatrix}$ and $A^2 = \begin{bmatrix}\alpha & \beta\\ \beta & \alpha\end{bmatrix}$, then find the value of $\propto and\ \beta$.

5. If A is a square matrix such that $A^2 = I$, then write the value of $(A - I)^3 + (A + I)^3 - 7$ A in simplest form.

6. Write the value of Δ, if

$$\Delta = \begin{vmatrix}x+y & y+z & z+x\\ z & x & y\\ -3 & -3 & -3\end{vmatrix}$$

7. If $\begin{bmatrix}x-y & z\\ 2x-y & w\end{bmatrix} = \begin{bmatrix}-1 & 4\\ 0 & 5\end{bmatrix}$, find the value of x+y.

8. If A is a 3 × 3 matrix, $|A| \neq 0$ and $|3A| = K|A|$, then write the value of K.

9. If $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is a symmetric matrix, then write the value of x.

10. Matrix $A = \begin{bmatrix} 0 & 2a & -2 \\ 3 & 1 & 3 \\ 3b & 3 & -1 \end{bmatrix}$ is given to be symmetric, find the value of a and b.

11. For any 2 × 2 matrix A, if A (adjoint A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then find $|A|$.

12. Find X, if A + X = I, where

$$A = \begin{bmatrix} 1 & 4 & -1 \\ 3 & 4 & 7 \\ 5 & 1 & 6 \end{bmatrix}$$

13. If $U = [2 \quad -3 \quad 4], V = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, X= $[0 \quad 2 \quad 3]$ and $Y = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$, then find UV+XY.

15. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then find the value of A^3.

16. Find the value of $a_{23} + a_{32}$ in the matrix

$A = [a_{ij}]_{3\times3}$ where $a_{ij} = \begin{cases} |2\mathrm{i} - \mathrm{j}| & \text{if i} > j \\ -\mathrm{i} + 2\mathrm{j} + 3 & \text{if i} < j \end{cases}$

17. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, then find $|A^2|$.

18. For what value of x, is the matrix

$$A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & x & -3 \\ 2 & 3 & 0 \end{bmatrix}$$ a skew-symmetric matrix

19. If $A = \begin{bmatrix} \sin 15^\circ & \cos 15^\circ \\ -\sin 75^\circ & \cos 75^\circ \end{bmatrix}$, then evaluate $|A|$.

20. If A is a square matrix, expressed as A= X + Y where X is symmetric and Y is skew-symmetric, then write the values of X and Y.

21. Write a matrix of order 3 × 3 which is both symmetric and skew-symmetric matrix.

22. What positive value of x makes the following pair of determinants equal?

$$\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}, \quad \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$$

23. $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, find the value of $5A_{31} + 3A_{32} + 8A_{33}$.

24. If $A = \begin{bmatrix} 2 & 1 \\ 7 & 5 \end{bmatrix}$, find $|A\ (adjA)|$

25. Find the minimum value of. $2\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+sin\theta & 1 \\ 1 & 1 & 1+cos\theta \end{vmatrix}$

26. If A and B are square matrices of order 3 and $|A| = 5$ and $|B| = 3$, then find the value of $|3AB|$.

27. Evaluate $\begin{vmatrix} 3+2i & -6i \\ 2i & 3-2i \end{vmatrix}$, $i = \sqrt{-1}$

---

29. Using determinants, find the equation of line passing through (0, 3) and (1,1).

30. If A be any square matrix of order 3 × 3 and $|A| = 5$, then find the value of $|adj\ (adjA)|$

31. What is the number of all possible matrices of order 2 × 3 with each entry 0,1 or 2.

32. Given a square matrix A of order 3 × 3 such that $|A|$=12, find the value of $|A\ adj\ A|$

33. If $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ find $|(A^{-1})^{-1}|$

34. If $A = [-1 \quad 2 \quad 3]$ and $B = \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$ find $|AB|$

35. Find $|A\ (adjoint\ A)|$ and $|adjoint\ A|$, if $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

TWO MARK QUESTIONS

1. Construct a matrix of order 2 × 3, whose elements are given by

(a) aij = $\frac{(i-2j)^2}{2}$ (b) aij = $\frac{|-2i+j|}{3}$

2. If A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are vertices of on equilateral triangle with each side equal to a units, than prove that

$$\begin{bmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{bmatrix}^2 = 3a^4$$

4. If $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

Find the valve of x – y

5. If A and B are skew symmetric matrices of the same order prove that AB + BA is symmetric matrix.

6. Without expending prove that $\begin{bmatrix} o & p-q & p-r \\ q-p & o & q-r \\ r-p & r-q & o \end{bmatrix} = 0$

7. Let A= $\begin{bmatrix} 2 & 5 \\ 4 & 6 \end{bmatrix}$ Prove that A+A′ is symmetric matrix.

8. If A= $\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$ and B=[1 2 3], Verify (AB)′ = B′A′

9. If A= $\begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}$, B= $\begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$ and C= $\begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

Find AB-AC.

10. If A= $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$ Find the determinant of A^2-2A

11. Without expanding, evaluate $\begin{bmatrix} 265 & 240 & 219 \\ 240 & 225 & 198 \\ 219 & 198 & 181 \end{bmatrix}$

12. If $D_1 = \begin{vmatrix} a & b & c \\ x & y & z \\ l & m & n \end{vmatrix}$ and $D_2 = \begin{vmatrix} m & -b & y \\ -l & a & -x \\ n & -c & z \end{vmatrix}$ evaluate D_1+D_2.

13. If A is a skew symmetric matrix of odd order, then prove that |A|= 0

14. Write the minors and co-factors of each element of the first column of the matrix A

A= $\begin{bmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{bmatrix}$

15. Find x and y, if $\begin{bmatrix} 2x+1 & 3y \\ 0 & y^2-5y \end{bmatrix} = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$

16. If A = $\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$, B= $\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$, find matrix 'C', such that 2A+3C=5B

17. If A = $\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ find x such that A^2 =B.

18. Construct a matrix of order 3x2, whose elements a_{ij} given by

$$a_{ij} = \begin{cases} 2i-3j, & i \geq j \\ 3i+j, & i < j \end{cases}$$

4 MARK QUESTIONS

5. If $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$, show that $A^2 - 12A - I = 0$. Hence find A^{-1}.

6. Find the matrix X so that $X\begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 2 & 0 \end{bmatrix}$

7. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, verify that $A^2 - 4A - 5I = 0$.

9. If $A = \begin{bmatrix} x & -2 \\ 3 & 7 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} \frac{7}{34} & \frac{1}{17} \\ -\frac{3}{34} & \frac{2}{17} \end{bmatrix}$, then find the value of x.

10. If $A = \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$, find B, such that $4A^{-1} + B = A^2$

11. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, 10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \propto \\ 1 & -2 & 3 \end{bmatrix}$ and B=A^{-1}, then find the value of $\propto$.

12. Find the value of X, such that $A^2 - 5A + 4I + X = 0$, if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

13. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, find $(A')^{-1}$

14. The monthly incomes of Mohan and Sohan are in the ratio 3:4 and their monthly expenditures are in the ratio 5:7. If each saves ₹ 15000/- per month, find their monthly incomes and expenditures using matrices.

15. If $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}$, then verify that (AB)′ = B′A′

16. If $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $x^2 = -1$

Then show that $(A + B)^2 = A^2 + B^2$

17. Prove that $aI + bA + cA^2 = A^3$, if $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & b & c \end{bmatrix}$

18. If $A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$, then find A^3.

19. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2 + 2AB$, find *a* and *b*.

20. If $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$, then find the value of a, b and c. Such that $A^T A = I$

21. If $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} a^n & b(\frac{a^n - 1}{a - 1}) \\ 0 & 1 \end{bmatrix}$, for all $n \in N$.

15. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

and use it to solve the system of equations:

$x - y + z = 4,\ x - 2y - 2z = 9,\ 2x + y + 3z = 1$

16. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and use it to solve the system of linear equations: $x + 2y + z = 4,\ -x + y + z = ,\ x - 3y + z = 2$

17. Solve given system of equations by matrix method:

$\frac{2}{a} + \frac{3}{b} + \frac{4}{c} = -3,\ \frac{5}{a} + \frac{4}{b} - \frac{6}{c} = 4,\ \frac{3}{a} - \frac{2}{b} - \frac{2}{c} = 6$

18. To raise money for an orphanage, students of three schools A, B and C organized an exhibition in their locality, where they sold paper bags, scrap books and pastel sheets made by them using recycled paper, at the rate of ₹ 20, ₹ 15 and ₹ 5 per unit respectively. School A sold 25 paper bags, 12 scrap books and 34 pastel sheets. School B sold 22 paper bags, 15 scrap books and 28 pastel sheets. While school C sold 26 paper bags, 18 scrap books and 36 pastel sheets. Using matrices, find the total amount raised by each school.

19. Two cricket teams honored their players for three values, excellent batting, to the point bowling and unparalleled fielding by giving ₹ x, ₹ y and ₹ z per player respectively. The first team paid respectively 2, 2 and 1 players for the above values with a total prize money of 11 lakhs, while the second team paid respectively 1,2 and 2 players for these values with a total prize money of ₹ 9 lakhs. If the total award money for one person each for these values amount to ₹ 6 lakhs, then express the above situation as a matrix equation and find award money per person for each value.

ANSWERS

ONE MARK QUESTIONS

1. $\frac{1}{2}$
2. $\lambda = 4$
3. $A^2 = I_3$
4. $\alpha = a^2 + b^2,\ \beta = 2ab$
5. A
6. 0
7. 3
8. K=27
9. X = 5
10. a=$\frac{3}{2}$, $b = \frac{-2}{3}$
11. $|A| = 10$
12. $X = \begin{bmatrix} 0 & -4 & 1 \\ -3 & -3 & -7 \\ -5 & -1 & -5 \end{bmatrix}$
13. [20]
14. $\begin{bmatrix} 2 & -3 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} -4 & -17 \\ 16 & 47 \end{bmatrix}$

15. $A^3 = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

16. 11

17. 0

18. $x = 0$

19. $|A| = 1$

20. $x = \frac{1}{2}(A + A'),\ y = \frac{1}{2}(A - A')$

21. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

22. $x = \pm 4$

23. 0

24. 9

25. –1

26. 405

27. 1

28. 0

29. 3 – 2x

30. 625

31. 729

32. 1728

33. 11

34. –11

35. $a^9,\ a^6$

2 MARK QUESTIONS

1 (a) $\begin{bmatrix} 1 & 9 & 25 \\ 2 & 2 & 2 \\ 0 & 2 & 8 \end{bmatrix}$

(b) $\begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{3} \\ 1 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

4. $x - y = -7$

9. $\begin{vmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{vmatrix}$

10. 25

11. 0

12. $D_1 + D_2 = 2$

14. $M_{11} = -12,\ M_{21} = -16,\ M_{31} = -4$
$C_{11} = -12,\ C_{21} = 16, C31 = -4$

15. x=2,y=2

16. $C = \begin{vmatrix} 12 & 4/3 \\ 4 & -14/3 \\ 25/3 & 28/3 \end{vmatrix}$

17. No value of x, for which $A^2=B$.

18. $A = \begin{vmatrix} -1 & 5 \\ 1 & -2 \\ 3 & 0 \end{vmatrix}$

4 MARKS QUESTION

2. $x = 0$

4. $-5\sqrt{3}(5-\sqrt{6})$

5. $A^{-1} = \begin{bmatrix} -7 & 3 \\ 12 & -5 \end{bmatrix}$

6. $\begin{bmatrix} 5 & 0 \\ -\frac{6}{7} & \frac{4}{7} \end{bmatrix}$

8. $\frac{1}{10}\begin{bmatrix} 7 & -1 \\ -4 & 2 \end{bmatrix}$

9. x = 4

10. $B = \begin{bmatrix} 2 & -15 \\ 0 & -3 \end{bmatrix}$

11. $\propto = 5$

12. $X = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{bmatrix}$

13. $\begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

14. Incomes: Rs 90,000/- and Rs 1,20,000/-

Expenditures: Rs 75,000/- and Rs 10,5000/-

18. $\begin{bmatrix} \cos 8\theta & \sin 8\theta \\ -\sin 8\theta & \cos 8\theta \end{bmatrix}$

19. a = – 1, b = – 2

20. $a = \pm\frac{1}{\sqrt{2}}; \quad b = \pm\frac{1}{\sqrt{6}};$

$c = \pm\frac{1}{\sqrt{3}}$

22. K = 2

23. x =2

24. K =(a + b) (b + c) (c + a)

32. $AB = \begin{bmatrix} 1 & 2 \\ -2 & 2 \end{bmatrix}$

$(AB)^{-1} = \frac{1}{6}\begin{bmatrix} 2 & -2 \\ 2 & -1 \end{bmatrix}$

33. x = 4

34. $x = \frac{-abc}{ab+bc+ca}$

35. $A = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$

36. x = 1,3

37. x=0,-12

39. ax (2a + 3x)

40. $A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

3. x = 0; y= -5; z = –3

4. $A^{-1} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$

5 $x = \begin{bmatrix} -16 & 3 \\ 24 & -5 \end{bmatrix}$

6. $\begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$

8. 0

13. β

15. *Product* = 81

x = 3, y = -2, z = -1

16. $A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ $x = \frac{9}{5}, y = \frac{2}{5}, z = \frac{7}{5}$

17. *a* = 1, *b* = – 1, *c* = – 2

18. School A = ₹ 850

School B = ₹ 805

School C = ₹ 970

19. Excellent batting: 3 lakhs

point bowling: 2 lakhs

fielding: 1lakh

20. $A^{-1} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

x=0, y=-5, z=-3.